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REAL ANALYSIS 2
FINAL EXAM SAMPLE PROBLEM SOLUTIONS

Sample Exam Problems:

(1) Be able to do the problems on the two previous tests.
OK.

(2) Prove that a function $f: X \rightarrow \mathbb{R}$ is measurable if and only if $f^{-1}((-\infty, t])$ is a measurable set for every $t \in \mathbb{R}$.

Proof: If f is measurable, then $f^{-1}((-\infty, y]) = f^{-1}((-\infty, y])$ is measurable for each $y \in \mathbb{R}$. Given any $t \in \mathbb{R}$, let

$$A_n = f^{-1}\left(\left(-\infty, t + \frac{1}{n}\right)\right),$$

which is measurable. Then

$$\bigcup_{n=1}^{\infty} A_n$$

is measurable. If $x \in \bigcup_{n=1}^{\infty} A_n$, then $f(x) < t + \frac{1}{n}$ for some $n \in \mathbb{N}$, so then $f(x) < t$, so $x \in f^{-1}((-\infty, t])$. On the other hand, if $x \in f^{-1}((-\infty, t])$, then $f(x) \leq t$, and since for $\epsilon = t - f(x)$, there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < \epsilon$. Thus $f(x) < t + \frac{1}{n}$, so $x \in A_n \subset \bigcup_{n=1}^{\infty} A_n$. Thus

$$f^{-1}((-\infty, t]) = \bigcup_{n=1}^{\infty} A_n$$

is measurable. Conversely, if $f^{-1}((-\infty, t])$ is measurable for every $t \in \mathbb{R}$, then for every $y \in \mathbb{R}$, the set

$$B_n = f^{-1}\left(\left(-\infty, y + \frac{1}{n}\right)\right)$$

is measurable, so that

$$\bigcap_{n=1}^{\infty} B_n$$

is measurable. If $x \in \bigcap_{n=1}^{\infty} B_n$, then $f(x) < y + \frac{1}{n}$ for every $n \in \mathbb{N}$, so by the order limit theorem $f(x) \leq y$, which implies that $x \in f^{-1}((-\infty, y]) = f^{-1}((-\infty, y])$. On the other hand, if $x \in f^{-1}((-\infty, y])$, then $f(x) \leq y + \frac{1}{n}$ for every $n \in \mathbb{N}$, so $x \in \bigcap_{n=1}^{\infty} B_n$. Thus

$$f^{-1}((-\infty, y]) = \bigcap_{n=1}^{\infty} B_n$$

is measurable for each $y \in \mathbb{R}$, so f is a measurable function. QED